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Non-linear mass transfer in boundary layers—
1. Asymptotic theory

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Abstract—An asymptotic theory of non-linear mass transfer in laminar gas–liquid systems with a moving phase interface is proposed. The problem is solved in the diffusion boundary layer approximation accounting for the interaction between the hydrodynamic flow and the diffusive flux for the case of intensive mass transfer between the gas and the liquid.

INTRODUCTION

INDUSTRIAL gas absorption is most often realized in packed bed columns. Having in mind that the particles of the bed are small, the interphase mass transfer of an absorbed substance takes place in thin layers, close to the moving phase boundary. In earlier studies [1] mass transfer in diffusive boundary layers has been studied from the point of view of linear diffusive boundary layer approximation theory.

It was shown [2] that in the presence of intensive mass transfer large mass fluxes can induce a secondary flow through the phase boundary. The velocity of this flow, v_n, which is normal to the phase interface, could be directly calculated [3] utilizing the diffusive mass flow through the interface

$$v_n = -\frac{MD}{\rho_0}\frac{\partial c}{\partial n}. \qquad (1)$$

Equation (1) is valid at the phase interface, while $\partial/\partial n$ denotes differentiation normal to the interface. Thus one finds that the mass flux through the phase interface comprises both diffusive and convective components [3]

$$I = -MD\frac{\partial c}{\partial n} + Mc^{*}v_n = -\frac{MD\rho^{*}}{\rho_0}\frac{\partial c}{\partial n} \qquad (2)$$

where

$$\rho^{*} = \rho_0 + Mc^{*}.$$

It follows directly from equation (1) that in the presence of large concentration gradients the convective diffusion equation is nonlinear. For the case of gas–liquid systems it was shown [4] that in the liquid these non-linear effects have to be taken into consideration only for the case of high pressure absorption. In the gas the non-linear effects are present in the majority of cases and could arise at normal pressures too.

Non-linear mass transfer in liquids has been theoretically analysed in refs. [1, 5] for electrochemical dissolving, ionic transfer and dissolving of the walls of a flat channel [1, 6].

In gases the non-linear mass transfer is connected to mass transfer induced Stefan-type flows [7]. Non-linear effects of this type have been theoretically studied in refs. [1, 8] for the case of condensation from a turbulent flow assuming a constant mass transfer rate along the condensate film surface.

The purpose of this paper is to report some theoretical results on non-linear mass transfer in a laminar gas–liquid system with a moving phase interface in the diffusion boundary layer approximation.

THE MATHEMATICAL MODEL

Time-independent momentum and mass transfer of the absorbed substance are described by a system of partial differential equations comprising the Navier–Stokes equations, the continuity and the convective diffusion equation. This system can be written in a convenient form [1] introducing the following dimensionless variables:

$$\tilde{u} = \frac{1}{2}\tilde{u}_{\infty}\varepsilon_1\phi_1'$$

$$\tilde{v} = \frac{1}{2}\left(\frac{\tilde{u}\tilde{\nu}}{x}\right)^{1/2}(\xi_1\phi_1' - \phi_1)$$

$$\tilde{c} = \tilde{c}_0 - \tilde{c}_0\psi_1$$

$$\phi_1 = \phi_1(\xi_1), \quad \psi_1 = \psi_1(\xi_1)$$

NOMENCLATURE

c	molar concentration of the absorbed substance
D	diffusivity
I	local mass transfer rate of the substance absorbed through the phase interface
k	mass transfer coefficient
M	molecular mass of the absorbed substance
u	velocity along the flow
v	velocity across the flow.

Greek symbols

μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
χ	Henry's constant.

Subscripts

0	beginning of the phase interaction or density in the absence of the absorbed substance
∞	far from the phase interface.

Superscripts

*	phase interface
~	gas.

$$\xi_1 = y\left(\frac{u_\infty}{4\tilde{D}x}\right)^{1/2}, \quad y \geqslant 0$$

$$\varepsilon_1 = \widetilde{Sc}^{1/2}, \quad \widetilde{Sc} = \frac{\tilde{\nu}}{\tilde{D}} \tag{3}$$

$$u = u_\infty \varepsilon_2 \phi_2'$$

$$v = -\left(\frac{n_\infty \nu}{\varkappa}\right)^{1/2} (\xi_2 \phi_2' - \phi_2)$$

$$c = \frac{\tilde{c}_0}{\chi}\psi_2$$

$$\phi_2 = \phi_2(\xi_2), \quad \psi_2 = \psi_2(\xi_2)$$

$$\xi_2 = -y\left(\frac{u_\infty}{4Dx}\right)^{1/2}, \quad y \leqslant 0$$

$$\varepsilon_2 = Sc^{1/2}, \quad Sc = \frac{\nu}{D}. \tag{4}$$

In a boundary layer approximation this system takes the form

$$\phi_1''' + \varepsilon_1^{-1}\phi_1\phi_1'' = 0$$
$$\psi_1'' + \varepsilon_1\phi_1\psi_1' = 0$$
$$\phi_2''' + 2\varepsilon_2^{-1}\phi_2\phi_2'' = 0$$
$$\psi_2'' + 2\varepsilon_2\phi_2\psi_2' = 0. \tag{5}$$

Equations (5) are subject to boundary conditions expressing the existence of a constant velocity potential flow far from the phase interface, cocurrent flows of the separate phases, continuity of the velocity, stress and the mass flux at the phase interface. It is also assumed that in both the liquid and the gas the concentrations satisfy Henry's law

$$\phi_1(0) = -\theta_3\psi_1'(0), \quad \phi_1'(0) = 2\theta_1\frac{\varepsilon_2}{\varepsilon_1}\phi_2'(0)$$

$$\phi_1'(\infty) = 2\varepsilon_1^{-1}$$

$$\psi_1(0) = 1 - \psi_2(0), \quad \psi_1(\infty) = 0$$

$$\phi_2(0) = \theta_4\psi_2'(0), \quad \phi_2'(\infty) = \varepsilon_2^{-1}$$

$$\phi_2''(0) = -\frac{1}{2}\theta_2\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2\phi_1''(0)$$

$$\psi_2(\infty) = 0, \quad \psi_2'(0) = \frac{\chi}{\varepsilon_0}\psi_1'(0) \tag{6}$$

where

$$\theta_1 = \frac{u_\infty}{\tilde{u}_\infty}, \quad \theta_2 = \frac{\tilde{\mu}}{\mu}\left(\frac{\tilde{\nu}}{\nu}\right)^{-1/2}\left(\frac{\tilde{u}_\infty}{u_\infty}\right)^{3/2}$$

$$\theta_3 = \frac{M\tilde{c}_0}{\tilde{\rho}_0\varepsilon_1}, \quad \theta_4 = \frac{M\tilde{c}_0}{2\rho_0\varepsilon_2\chi}$$

$$\varepsilon_0 = \left(\frac{u_\infty D\rho^*\tilde{\rho}_0}{\tilde{u}_\infty\tilde{D}\tilde{\rho}^*\rho_0}\right)^{1/2}. \tag{7}$$

Parameters θ_1 and θ_2 account for the kinematic and the dynamic interactions between the separate phases while θ_3 and θ_4 are responsible for the influence of the mass transfer on the hydrodynamics (having in mind equation (1) for the gas and the liquid phases).

From equations (5) and (6) it follows directly that the concentration of the absorbed substance at the phase interface is constant; therefore new boundary conditions can be introduced, namely

$$\psi_1(0) = A, \quad \psi_2(0) = 1 - A \tag{8}$$

where A can be determined utilizing the last of equations (6). Thus, equations (5) could be solved as two separate systems.

THE METHOD OF SOLUTION

For cases of practical interest the parameters θ_i, $i = 1, 2, 3, 4$, can be considered to be small and equations (5) and (6) could be solved by perturbation methods, expressing the unknown functions by expansion of the type

$$F = F^{(0)} + \theta_1 F^{(1)} + \theta_2 F^{(2)} + \theta_3 F^{(3)} + \theta_4 F^{(4)}. \tag{9}$$

If the corresponding expressions of the form of equation (9) are introduced in equations (5) and (6), one can obtain separate equations to determine the new functions.

The zeroth-order approximation follows directly from equations (5) and (6) if one sets $\theta_i = 0$, $i = 1, 2, 3, 4$.

First-order approximations are obtainable from

$$\begin{gathered}\phi_j^{(i)'''} + j\varepsilon_j^{-1}(\phi_j^{(i)''}\phi_j^{(0)} + \phi_j^{(0)''}\phi_j^{(i)}) = 0\\ \psi_j^{(i)''} + j\varepsilon_i(\phi_j^{(i)}\psi_j^{(0)'} + \phi_j^{(0)}\psi_j^{(i)'}) = 0\\ i = 1, 2, 3, 4 \quad j = 1, 2\end{gathered} \tag{10}$$

subject to boundary conditions

$$\begin{aligned}\phi_j^{(i)}(0) &= 0, \quad i = 1, 2, \quad j = 1, 2\\ \phi_1^{(3)}(0) &= -\psi_1^{(0)'}(0), \quad \phi_2^{(4)}(0) = \psi_2^{(0)'}(0)\\ \phi_1^{(i)'}(0) &= 0, \quad i = 2, 3, 4\\ \phi_1^{(1)}(0) &= 2(\varepsilon_2/\varepsilon_1)\phi_2^{(0)'}(0)\\ \phi_j^{(i)'}(0) &= 0, \quad i = 1, 2, 3, 4, \quad j = 1, 2\\ \psi_1^{(i)}(0) &= A^{(i)}, \quad i = 1, 2, 3, 4\\ \psi_j^{(i)}(\infty) &= 0, \quad i = 1, 2, 3, 4, \quad j = 1, 2\\ \phi_1^{(4)}(0) &= 0, \quad \phi_2^{(3)}(0) = 0\\ \phi_2^{(i)''}(0) &= 0, \quad i = 1, 3, 4\\ \phi_2^{(2)}(0) &= -\tfrac{1}{2}(\varepsilon_1/\varepsilon_2)^2\phi_1^{(0)''}(0)\\ \psi_2^{(i)}(0) &= -A^{(i)}.\end{aligned} \tag{11}$$

The values for $A^{(i)}$ ($i = 1, 2, 3, 4$) can be calculated from

$$\psi_2^{(i)}(0) = \frac{\chi}{\varepsilon_0}\psi_1^{(i)'}(0), \quad i = 1, 2, 3, 4. \tag{12}$$

THE ZEROTH-ORDER APPROXIMATION

The zeroth-order approximation has been reported in a number of papers [9–13]

$$\begin{aligned}\phi_1^{(0)}(\xi_1) &= f(z), \quad z = \frac{2}{\varepsilon_1}\xi_1\\ \psi_1^{(0)}(\xi_1) &= A^{(0)}\left\{1 - \frac{1}{\varphi_0}\int_0^z \exp\left[-\frac{\varepsilon_1^2}{2}\int_0^p f(s)\,\mathrm{d}s\right]\mathrm{d}p\right\}\\ \phi_2^{(0)}(\xi_2) &= \varepsilon_2^{-1}\xi_2\\ \psi_2^{(0)}(\xi_2) &= (1 - A^{(0)})\operatorname{erfc}\xi_2\\ A^{(0)} &= \frac{1}{1+a}, \quad a = \frac{\chi\sqrt{\pi}}{\varepsilon_0\varepsilon_1\varphi_0}\end{aligned} \tag{13}$$

where $f(z)$ is the solution of the boundary value problem

$$2f''' + ff'' = 0, \quad f(0) = f'(0) = 0, \quad f'(\infty) = 1 \tag{14}$$

and it is tabulated in ref. [10], while φ_0 is a function of the Schmidt number for the gas

$$\varphi_0 = \int_0^\infty \exp\left[-\frac{\widetilde{Sc}}{2}\int_0^p f(s)\,\mathrm{d}s\right]\mathrm{d}p. \tag{15}$$

Its values are tabulated in ref. [11] or could be calculated by approximate formulas [1].

THE KINEMATIC INTERACTION

The kinematic interaction between the phases is a result from the continuity of the velocity distribution at the phase interface and it is accounted for by θ_1 and the following functions (some of which have been obtained earlier [9–13]):

$$\begin{aligned}\phi_1^{(1)}(\xi_1) &= \frac{1}{\alpha}f'(z), \quad \phi_2^{(1)}(\xi_2) \equiv 0\\ \psi_1^{(1)}(\xi_1) &= A^{(1)} + \frac{A^{(0)}}{\alpha\varphi_0}\left\{1 - \exp\left[-\frac{\varepsilon_1^2}{2}\int_0^z f(s)\,\mathrm{d}s\right]\right\}\\ &\quad - \left(\frac{A^{(1)}}{\varphi_0} + \frac{\varepsilon_1 A^{(0)}}{2\alpha\varphi_0^2}\right)\int_0^z \exp\left[-\frac{\varepsilon_1^2}{2}\int_0^p f(s)\,\mathrm{d}s\right]\mathrm{d}p\\ \psi_2^{(1)}(\xi_2) &= -A^{(1)}\operatorname{erfc}\xi_2\\ A^{(1)} &= -\frac{\varepsilon_1}{2\alpha\varphi_0}\frac{a}{(1+a)^2}.\end{aligned} \tag{16}$$

THE DYNAMIC INTERACTION

The dynamic interaction between the phases is a result of the continuity of the stresses at the phase interface. It is accounted for by θ_2 and the following functions (some of which have been obtained earlier [9–11]):

$$\phi_1^{(2)}(\xi_1) \equiv 0, \quad \phi_2^{(2)}(\xi_2) = \alpha\sqrt{\pi}\int_0^{\xi_2/\varepsilon_2}\operatorname{erfc}p\,\mathrm{d}p$$

$$\psi_1^{(2)}(\xi_1) = A^{(2)}\left\{1 - \frac{1}{\varphi_0}\int_0^z \exp\left[-\frac{\varepsilon_1^2}{2}\int_0^p f(s)\,\mathrm{d}s\right]\mathrm{d}p\right\}$$

$$\begin{aligned}\psi_2^{(2)}(\xi_2) = {}& -A^{(2)} + [A^{(2)} - 4\alpha\varepsilon_2(1 - A^{(0)})\varphi_2]\operatorname{erf}\xi_2\\ &+ 4\alpha\varepsilon_2(1 - A^{(0)})Q(Sc, \xi_2)\end{aligned}$$

$$\begin{aligned}Q(Sc, \xi_2) &= \int_0^{\xi_2}\left\{\exp(-q^2)\int_0^q \right.\\ &\qquad \left.\times\left[\int_0^{p/\varepsilon_2}\operatorname{erfc}s\,\mathrm{d}s\right]\mathrm{d}p\right\}\mathrm{d}q\\ &= \frac{\sqrt{\pi}}{8\varepsilon_2}\operatorname{erf}\xi_2 - \frac{1}{4\varepsilon_2}\xi_2\exp(-\xi_2^2)\\ &\quad - \frac{2}{\sqrt{\pi}}\sum_{i=0}^{\infty}\frac{(-1)\varepsilon_2^{-2i-2}Q_i}{i!(2i+1)(2i+2)(2i+3)}\end{aligned}$$

$$Q_i = \int_0^{\xi_2} \mathrm{e}^{-q^2}q^{2i+3} = -\tfrac{1}{2}\xi_2^{2i+2}\exp(-\xi_2^2) + (i+1)Q_{i-1}$$

$$Q_0 = \tfrac{1}{2} - \tfrac{1}{2}\exp(-\xi_2^2) - \tfrac{1}{2}\xi_2^2\exp(-\xi_2^2)$$

Table 1

$Sc^{1/2}$	$Q(Sc, \infty)$	$\varphi_2 = (1/8)\sqrt{(\pi/Sc)}$
1.1	0.14519	0.20142
1.2	0.14421	0.18463
1.5	0.13053	0.14770
2.0	0.10517	0.11078
5.0	0.04416	0.04431
10.0	0.02216	0.02216
20.0	0.01108	0.01108
50.0	0.00443	0.00443
100.0	0.00222	0.00222

$$A^{(2)} = 4\alpha\varepsilon_2\varphi_2\frac{a}{(1+a)^2}$$

$$\varphi_2 = \varphi_2(Sc) = Q(Sc, \infty). \tag{17}$$

Having in mind that $\varepsilon_2 > 10$ for Q and φ_2 one could assume with acceptable accuracy

$$Q(Sc, \xi_2) = \frac{1}{8}\sqrt{\left(\frac{\pi}{Sc}\right)}\operatorname{erf}\xi_2 - \frac{1}{4}\frac{1}{\sqrt{Sc}}\xi_2\exp(-\xi_2^2)$$

$$\varphi_2(Sc) = \frac{1}{8}\sqrt{\left(\frac{\pi}{Sc}\right)}. \tag{18}$$

Table 1 presents the values of $\varphi_2(Sc)$. In the second column are listed the values of this function from $Q(Sc, \infty)$, equations (17). These quantities were calculated utilizing the series expansion for the error function, substituting it in the integral, and calculating the sum of the series obtained after the integration. In the last column of Table 1 one finds the values of φ_2 as calculated from the last of equations (18).

THE NON-LINEAR MASS TRANSFER EFFECTS IN THE GAS

The non-linear mass transfer effects in the gas are a result of the Stefan type flow. It is accounted for by θ_3 and the functions

$$\phi_1^{(3)}(\xi_1) = \frac{2A^{(0)}}{\varepsilon_1\varphi_0}\varphi(z), \quad \phi_2^{(3)}(\xi_2) \equiv 0$$

$$\psi_1^{(3)}(\xi_1) = A^{(3)} - \left(\frac{A^{(3)}}{\varphi_0} + \frac{\varepsilon_1 {A^{(0)}}^2\varphi_3}{\varphi_0^3}\right)\int_0^z \exp$$

$$\times\left[-\frac{\varepsilon_1^2}{2}\int_0^p f(s)\,\mathrm{d}s\right]\mathrm{d}p + \frac{\varepsilon_1 {A^{(0)}}^2}{\varphi_0^2}\int_0^z\left[\int_0^p \varphi(s)\,\mathrm{d}s\right]$$

$$\times\exp\left[-\frac{\varepsilon_1^2}{2}\int_0^p f(s)\,\mathrm{d}s\right]\mathrm{d}p$$

$$\psi_2^{(3)}(\xi_2) = -A^{(3)}\operatorname{erfc}\xi_2$$

$$A^{(3)} = -\frac{\varepsilon_1\varphi_3}{\varphi_0^2}\frac{a}{(1+a)^3} \tag{19}$$

where $\varphi(z)$ is the solution of the problem

$$2\varphi''' + f\varphi'' + f''\varphi = 0, \quad 2f''' + ff'' = 0$$

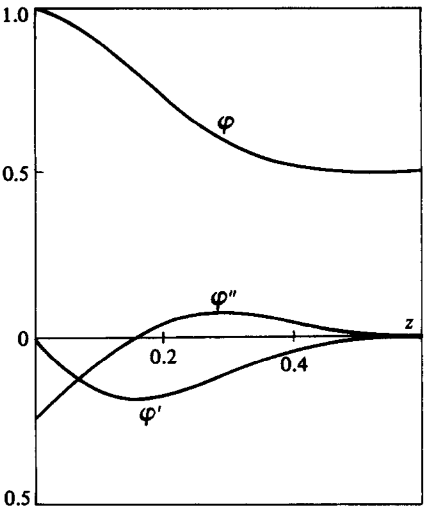

FIG. 1.

Table 2

$\widetilde{Sc}$	φ_3	$3.54\widetilde{Sc}^{0.67}$
0.10	17.49	16.42
0.20	10.40	10.34
0.50	5.46	5.61
0.80	3.99	4.10
1.00	3.45	3.54
1.20	3.06	3.13
1.50	2.65	2.70
1.80	2.36	2.39
2.00	2.21	2.23
5.00	1.24	1.21
10.00	0.80	0.76

$$\varphi(0) = 1, \quad \varphi'(0) = \varphi'(\infty) = 0, \quad f(0) = f'(0) = 0,$$
$$f'(\infty) = 1. \tag{20}$$

In equations (19) φ_3 is a function of the Schmidt number for the gas

$$\varphi_3 = \int_0^\infty\left[\int_0^p \varphi(s)\,\mathrm{d}s\right]\exp\left[-\frac{\widetilde{Sc}}{2}\int_0^p f(s)\,\mathrm{d}s\right]\mathrm{d}p. \tag{21}$$

Function φ is shown on Fig. 1, while the values of φ_3 are listed in Table 2. In the last column of this table one can find values of φ_3 calculated by an approximate formula.

THE NON-LINEAR MASS TRANSFER EFFECT IN THE LIQUID

The non-linear mass transfer effect in the liquid is due to mass transfer induced secondary flows in this phase. This effect is accounted for by θ_4 and the following functions:

$$\phi_1^{(4)}(\xi_1) \equiv 0, \quad \phi_2^{(4)}(\xi_2) = -\frac{2}{\sqrt{\pi}}(1 - A^{(0)})$$

$$\psi_1^{(4)}(\xi_1) = A^{(4)}\left\{1 - \frac{1}{\varphi_0}\int_0^z \exp\left[-\frac{\varepsilon_1^2}{2}\int_0^p f(s)\,\mathrm{d}s\right]\mathrm{d}p\right.$$

$$\psi_2^{(4)}(\xi_2) = -A^{(4)} - \frac{4\varepsilon_2}{\pi}(1-A^{(0)})^2 + \frac{4\varepsilon_2}{\pi}(1-A^{(0)})^2$$

$$\times \exp(-\xi_2^2) + \left[A^{(4)} + \frac{4\varepsilon_2}{\pi}(1-A^{(0)})^2\right]\operatorname{erf}\xi_2$$

$$A^{(4)} = -\frac{4\varepsilon_2}{\pi}\frac{a^2}{(1+a)^3}. \qquad (22)$$

THE DIMENSIONLESS VELOCITY PROFILES

The dimensionless velocity profiles are directly obtainable from equations (3), (4), (13), (16), (17), (19) and (22) in the form

$$\frac{\tilde{u}}{u_\infty} = f'(z) + \frac{\theta_1}{\alpha}f''(z) + \frac{2\theta_3}{\varepsilon_1\varphi_0}\frac{1}{1+a}\varphi'(z) \qquad (23)$$

$$\frac{u}{u_\infty} = 1 + \theta_2\alpha\sqrt{\pi}\operatorname{erfc}\frac{\xi_2}{\varepsilon_2} - \frac{8\theta_4\varepsilon_2}{\pi}\frac{a^2}{(1+a)^2}$$

$$\times\left(\xi_2 - \frac{1}{\sqrt{\pi}}\frac{a}{1+a}\right)\exp(-\xi_2^2). \qquad (24)$$

INTERPHASE MASS TRANSFER

The rate of the interphase mass transfer between the diffusive boundary layers of length l defined as

$$J = M\tilde{k}\tilde{c}_0 = Mk\frac{\tilde{c}_0}{\chi} \qquad (25)$$

can be expressed by means of the average mass flux through the phase interface as

$$J = \frac{1}{l}\int_0^l \tilde{I}\,\mathrm{d}x = \frac{1}{l}\int_0^l I\,\mathrm{d}x. \qquad (26)$$

One can substitute equations (2) into equation (26) for the gas and the liquid to obtain expressions for the Sherwood numbers

$$\tilde{S}h = \frac{\tilde{k}l}{\tilde{D}} = \frac{\tilde{\rho}^*}{\tilde{\rho}_0\tilde{c}_0}\int_0^l\left(\frac{\partial\tilde{c}}{\partial y}\right)_{y=0}\mathrm{d}x = -\frac{\tilde{\rho}^*}{\tilde{\rho}_0}\tilde{P}e^{1/2}\psi_1'(0)$$

$$Sh = \frac{\tilde{k}l}{D} = \frac{\rho^*\chi}{\rho_0\tilde{c}_0}\int_0^l\left(\frac{\partial c}{\partial y}\right)_{y=0}\mathrm{d}x = -\frac{\rho^*}{\rho_0}Pe^{1/2}\psi_2'(0)$$

$$(27)$$

where the Peclet numbers are defined as

$$\tilde{P}e = \frac{\tilde{u}_\infty l}{\tilde{D}}, \quad Pe = \frac{u_\infty l}{D} \qquad (28)$$

and for $\psi_1'(0)$ and $\psi_2'(0)$, having in mind equations (13)–(22), one has

$$\psi_1'(0) = -\frac{2}{\varepsilon_1\varphi_0}\frac{1}{1+a} - \frac{\theta_1}{\alpha\varphi_0^2}\frac{1}{(1+a)^2}$$

$$-8\theta_2\alpha\frac{\varepsilon_2}{\varepsilon_1}\frac{\varphi_2}{\varphi_0}\frac{a}{(1+a)^2} - 2\theta_3\frac{\varphi_3}{\varphi_0^3}\frac{1}{(1+a)^3}$$

$$+8\theta_4\frac{\varepsilon_2}{\pi\varphi_0\varepsilon_1}\frac{a^2}{(1+a)^3}$$

$$\psi_2'(0) = -\frac{2}{\sqrt{\pi}}\frac{a}{1+a} - \theta_1\frac{\varepsilon_1}{\sqrt{\pi\alpha\varphi_0}}$$

$$-8\theta_2\frac{\alpha\varepsilon_2\varphi_2}{\sqrt{\pi}}\frac{a^2}{(1+a)^2} - 2\theta_3\frac{\varepsilon_1\varphi_3}{\sqrt{\pi\varphi_0^2}}\frac{a}{(1+a)^3}$$

$$+8\theta_4\frac{\varepsilon_2}{\pi^{3/2}}\frac{a^3}{(1+a)^3}. \qquad (29)$$

When the rate of the interphase mass transfer is limited by the diffusion resistance in the gas, from the last of equations (6) it follows directly that $\chi/\varepsilon_0 \to 0$, that is $a \to 0$ and for the Sherwood number one has

$$\tilde{S}h = \frac{\tilde{\rho}^*}{\tilde{\rho}_0}\tilde{P}e^{1/2}\left(\frac{2}{\varepsilon_1\varphi_0} + \frac{\theta_1}{\alpha\varphi_0^2} + 2\theta_3\frac{\varphi_3}{\varphi_0^3}\right). \qquad (30)$$

When the process is limited by the resistance of the liquid phase $\chi/\varepsilon_0 \to \infty$, $a \to \infty$, therefore

$$Sh = \frac{\rho^*}{\rho_0}Pe^{1/2}\left(\frac{2}{\sqrt{\pi}} + 8\theta_2\frac{\alpha\varepsilon_2\varphi_2}{\sqrt{\pi}} - 8\theta_4\frac{\varepsilon_2}{\pi^{3/2}}\right). \qquad (31)$$

CONCLUSIONS

The analysis of equations (27)–(31) implies that the non-linear mass transfer effect shows up both through $\tilde{\rho}^*/\tilde{\rho}_0$ and ρ^*/ρ_0

$$\frac{\tilde{\rho}^*}{\tilde{\rho}_0} = 1 + \frac{M\tilde{c}^*}{\tilde{\rho}_0}, \quad \frac{\rho^*}{\rho_0} = 1 + \frac{Mc^*}{\rho_0}, \quad \tilde{c}^* = \chi c^* \qquad (32)$$

and θ_3 and θ_4

$$\theta_3 = \frac{C}{\varepsilon_1}, \quad \theta_4 = \frac{C}{2\varepsilon_2\chi}\frac{\tilde{\rho}_0}{\rho_0}, \quad C = \frac{M\tilde{c}_0}{\tilde{\rho}_0}. \qquad (33)$$

For low solubility gases ($\varepsilon_0/\chi \approx 0$) the effects of the kinematic interaction between the phases and the Stefan type flow in the gas are absent, while the non-linear mass transfer effect in the liquid depends on the pressure and it can be observed only when the pressure is high.

For high solubility gases ($\chi/\varepsilon_0 \approx 0$) the effects of the dynamic interaction between the phases and the non-linear mass transfer in the liquid are absent (although for this case $\chi \ll 1$ and it is possible that $\theta_4 > 10^{-2}$ even at normal pressure).

For moderately soluble gases ($10^{-1} < \varepsilon_0/\chi < 10$) non-linear mass transfer effects in the liquids are practically absent, because in this case $\theta_4 \approx 0$. But the Stefan type flow in the gas influences the mass transfer in both phases.

Non-linear mass transfer in either phase (the intensive mass transfer induced flow) does not influence the hydrodynamics in the other phase. This is obviously valid for the approximations discussed in the above, that is in the first approximation in θ_3 and θ_4.

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TRANSFERT DE MASSE NON LINEAIRE DANS DES COUCHES LIMITES—1. THEORIE ASYMPTOTIQUE

Résumé—On analyse l'influence des trajectoires des gouttelettes et de leur distribution en taille sur le mécanisme du transfert thermique à partir d'un dièdre chaud isotherme dans un écoulement air–brouillard d'eau. Pour simplifier les calculs, on utilise une méthode approchée avec une théorie du diamètre équivalent. Les équations de l'écoulement du film d'eau sont résolues par une méthode d'intégration, tandis que les équations de l'écoulement gazeux sont traitées par une méthode de différence finie. Les conditions d'interface entre film d'eau et couche de gaz sont déterminées par itération des solutions des deux écoulements d'eau et de gaz.

NICHTLINEARER MASSENTRANSPORT IN GRENZSCHICHTEN—1. ASYMPTOTISCHE THEORIE

Zusammenfassung—Es wird eine asymptotische Theorie des nichtlinearen Massentransports in laminaren Gas–Flüssigkeits-Systemen mit bewegter Phasengrenze vorgeschlagen. Das Problem wird mit einer Näherung der Diffusionsgrenzschichten gelöst unter Beachtung der Hydrodynamik und der Diffusionsstromdichte im Falle intensiven Massentransports zwischen Gas und Flüssigkeit.

НЕЛИНЕЙНЫЙ МАССООБМЕН В ПОГРАНИЧНЫХ СЛОЯХ—1. АСИМПТОТИЧЕСКАЯ ТЕОРИЯ

Аннотация—Предложена асимптотическая теория нелинейного массообмена в ламинарных системах газ–жидкость с движущейся межфазной поверхностью. Задача решалась в приближении диффузионного пограничного слоя, учитывающем взаимодействие между гидродинамическим и диффузионным потоками для случая интенсивного массообмена между газом и жидкостью.